**Distributionally Robust SAA for Insurance Portfolio with…**

**Objective**

I want to design and evaluate strategic asset allocations for insurance portfolios using distributionally robust optimization. The model integrates forward-looking capital market assumptions and institutional constraints to reflect real-world decision-making. The benchmark will be traditional mean-variance SAA model, but since it lacks robustness under distributional uncertainty and fails to capture the regulatory, rating, liquidity, and liability-matching needs of insurers, the DRO model will be evaluated with these constraints and objectives. This project will simulate and compare optimal portfolios aligned with different institutional objectives, using a consistent market and risk framework.

**Methodology**

**1. Capital Market Assumptions (CMAs)**

* **Covariance Matrix**: Historical estimates (stable over time).
* **Expected Returns**:
  + **Equities**: bottom-up, sector, size, geography dimensions, equity risk premium, quantitative + qualitative view
  + **Fixed Income**: YTM + roll-down − credit loss − duration impact.
  + **Alternatives**: private equity from public equity; RE, HF, and commodities (have to think about it)

**2. Optimization Framework**

* [**Distributionally Robust Optimization**](https://pdf.sciencedirectassets.com/271589/1-s2.0-S0307904X22X00139/1-s2.0-S0307904X22006199/main.pdf?X-Amz-Security-Token=IQoJb3JpZ2luX2VjEEcaCXVzLWVhc3QtMSJHMEUCIQCEaSW2B2ygDgm7CCjX4TvuL24Zx25xVWlltCygwcyo4gIgD2CKwq9jYeRNIzgmfGF2UVlZjpnCaV2jFs%2FU6alFnfQqsgUIHxAFGgwwNTkwMDM1NDY4NjUiDNP4iQxIrv7QkFxCYSqPBaPUW5iq%2FRccwLEdzK136E0T%2B3HdYDyXpUQQaIrEpftqtv2rSkznUjCpnyc3C8xogMqz34WWeo85CBsWqFqNajFaZPWubT2uy786bPSac%2FWQBqyFGa2%2FS98plGzJJwAr%2BIXmQwh3Kmk7woKs3i%2BUXu0TmlLHSNTDdkLuqkovDPfmw3JBVH4A8DMBaqoSt0f%2BbZ8Ar97iPsYZ0UZiB09WGPhxMQnxFf%2BzAVRfspa6Hoj2CBqv%2FMUk5Br962b3fVdqtt5C4SHrxLmFVyhsVT8GEWX1qKPpndX3gi3mdnx4q5e1PPmLtJ%2BNqLXVlPHTrLd22aHFsyw0WZ68Cy2TW3Dcgig9xCcZt97nWpL%2FnXr%2FAVey%2FQk2NFbpDXXSjuJAJpyjp2KdNq7nw7FPH6iMNFRjfGWWml%2BToFAxC0ehzY1uQ86LdgHYBPVkoGDeVVjy5HJsAsKqfJuh64O7JxxpUv3njkufdl%2FF9K4ALgtFOiA%2Bt%2FDmzzHZ7d5vI1jL6apt0Z2T2d4Nme1sWTkIonpXEGh4uW8qvUcHf4GSUlsvNuAZnn523EqBZjxizluOMZOpzcI69GNlaGHiKwV%2Bq4JAKtoDGMMynEKJnRvdc%2B5FMdZ8ddu2%2BB38Ak3CnBJ1ZHhSrhncGuNBAPYPs8XcsHEV3%2BQuJ7%2BubNkaU%2Fe4HYVKzmvKtwu3r8x3VBvOQWxPR13AaXCBfSdshqn9QTh9eQcMJ7auLH7Y%2B9ehNpdcx5pjZ1J1ZCRKO7nv0YFHzomBWmw3VPLkFnBcP02y%2FqGMeG4uL2L3OL%2BttswlB1xk1G%2FkLIyH7sGl8%2BF6OOYJUmz%2BGyy5QL4adAQzzJKO1uyuF4x9n%2FajgCnrKaSWmS84dH7iEOmhNz8wv%2Bj9wQY6sQE8jUVBtzSOXFMB8ZdShkpNTnI3NybeXpBCLctTOMETjG3L1D31McX892HxzBDEdxc2O97TLvSEV7N0qVJ9TxsuHBsSe9eo95UvWc5WFzjuCGjPIz1L5J0nBROOtEnnxjrFKGulz%2BCKNNCi8o8QSGDnhbK5NXm6TPWjKRO4a4tyxaj6YpPwcNCZtNlPZArKVStLNk0gGH2nW5V3ecxhAq%2BPZ8PN8RJ3h7ikH16OCCgplMg%3D&X-Amz-Algorithm=AWS4-HMAC-SHA256&X-Amz-Date=20250603T233527Z&X-Amz-SignedHeaders=host&X-Amz-Expires=300&X-Amz-Credential=ASIAQ3PHCVTYSHMFRCO5%2F20250603%2Fus-east-1%2Fs3%2Faws4_request&X-Amz-Signature=2e560074553755d2715191c52b5c644059f67989e9962ff83187d03374b29599&hash=3f52c80fd197a0c1a08fcf1ded3651498caa44b2e67298bfa497493e95a11e9d&host=68042c943591013ac2b2430a89b270f6af2c76d8dfd086a07176afe7c76c2c61&pii=S0307904X22006199&tid=spdf-e672b7e6-87b8-4bb8-bb32-0d3ed2a47377&sid=64e34d9251d9464aa92afe03e15491527e1fgxrqa&type=client&ts) using ambiguity sets (Wasserstein or moment bounds).

**Objective Function:**

**Maximize Expected Surplus – maxw minP∈U E(P[wᵀ r − l])**  
where w = portfolio weights, r = asset returns, l = liabilities, and U = ambiguity set.

**Constraints**

* Budget constraint
* No short-selling or capped exposures
* Liquidity floor
* Duration match with liabilities
* Minimum Net Investment Income (NII)
* Regulatory solvency constraints – RBC < X% (shrinks equity allocation)

**3. Return Scenario Generator: Multivariate GBM Engine**

**3.1 Equities (and listed REITs)**  
Multivariate geometric Brownian motion (monthly step Δt = 1 ⁄ 12):

ln S\_j(t+Δt) = ln S\_j(t) + (μ\_j − 0.5 σ\_j²) Δt + σ\_j √Δt · Z\_{j,t}

where **Z\_t ∼ 𝒩(0, Σ)** gives the desired cross‑asset correlations.

**3.2 Fixed Income (Govt & IG/HY Credit)**  
Short‑rate **Vasicek model** for the term structure:

* dr(t) = a(b − r(t))dt + σ\_r\*dW\_r(t)
* Zero‑coupon bond price:

P(t,T) = A(t,T) · exp(−B(t,T) · r(t))

* B(t,T) = (1 − e^{−a·(T−t)}) / a
* A(t,T) = exp{ (B−T+t)(a²b − 0.5 σ\_r²)/a² − 0.25 σ\_r² B²/a }
* Index‑level total‑return approximation (duration–convexity):

ΔP / P ≈ −D · Δy + 0.5 · C · (Δy)² + Roll\_down − Credit\_Loss

* where Δy = Δr + Δs separates risk‑free and spread shocks.

**3.3 Alternatives (Private Equity, Infrastructure, Real Estate)**  
Quarterly capital‑call / distribution engine:

* C\_t = Commitment · c(t)
* D\_t = NAV\_{t−1} · d(t)
* NAV\_t = NAV\_{t−1} · (1 + r\_alt,t) + C\_t − D\_t
* with vintage curves **c(t), d(t)** and smoothed NAV return
* r\_alt,t ∼ 𝒩(μ\_alt, σ\_alt).

**3.4 Unified Scenario Cube**  
Concatenate all simulated paths into a single tensor

* shape = (N\_scenarios (10,000), N\_steps (120months), N\_assets (10-15))
* so every scenario contains a coherent set of equity, bond, and alternative outcomes.

**Assessment Framework**

**Core Metrics**

* Expected Surplus
* Surplus Volatility
* Sharpe Ratio
* Expected shortfall
* Solvency Ratio
* NII Coverage Ratio

**Constraint Diagnostics**

* Liquidity Ratio
* Regulatory Capital Requirement
* Duration Gap
* Credit Risk Exposure
* Concentration Index

**Timeline**

* June: finalize the topic and scope, retrieve data, start building a model
* July: implement optimizers, calibrate constraints, and run simulations, evaluate, visualize, and write report